

# Charge conservation and Equivalence principle

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## Abstract

The  $TH\epsilon\mu$  formalism was developed to study nonmetric theories of gravitation. In this letter we show that theories that violate Local Lorentz Invariance (LLI) or Local Position Invariance (LPI) also violate charge conservation. Using upper bounds on this violation we can put very stringent limits to violations of Einstein Equivalence Principle (EEP). These limits, in turn, severely restrict string-based models of low energy physics.

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There are two theoretical frameworks which stand as the milestones of modern physics: the standard model of special relativistic particle physics, and general relativity as the gravitational theory. The former rests on gauge invariance symmetry, while the latter is built geometrically from EEP. Two observational testable laws follow from them: charge conservation (global gauge symmetry) and the weak equivalence principle (invariance of non gravitational laws in locally inertial frames). The local aspects of both schemes can change dramatically if there are long range interactions whose dynamics cannot be influenced in local experiments. One can even expect that local-frame global gauge invariance (not the same as local gauge invariance) as well as EEP may be violated, even if the complete (global) theory satisfy the invariances mentioned above.

Regarding possible violations of EEP, a scheme was developed at the beginning of the seventies ([1]) in order to analyze non metric gravitational theories in spherically symmetric static situations. This theoretical scheme, called the  $TH\epsilon\mu$  formalism, has also been used to “prove” Schiffs conjecture [2]. By non metric we mean theories that present long range fields (gravitation like fields) that couple with matter directly, besides the metric (which may still account for part of the gravitational sector of the theory). For instance, if there is a scalar field with long range interactions that couple directly with matter, then in a local falling frame, where the metric reduces to its Minkowskian form, we may still have time- or space-dependent factors in the local dynamics, which could render a non relativistic invariant local lagrangian. Any “external structure” (e.g. Minkowski-metric external structure replaced by the [dynamical] metric in general relativity [3]) such as the fundamental constants can be suspected of hiding long range fields that have frozen at some value, making the fundamental parameters effectively constant. Unification schemes such as superstring theories [4] and Kaluza-Klein theories [5] have cosmological solutions in which the low energy fundamental constants are functions of time (including possibly the speed of light [6, 7]), thus violating LLI and Local Position Invariance (LPI).

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Usually low-energy phenomena are used to constrain the variation rate of fundamental constants [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. It is well known that objects with space- or time-dependent masses follow paths which do not correspond to geodesics of the space-time metric [18]. Indeed this violations of LPI induce violations of the universality of free fall, thus being subject to very stringent tests. This tests are Eötvös-type experiments, in which the accelerations of neutral masses with different composition in the same gravitational field are compared (null gravitational acceleration experiments). These are the most precise tests of the equivalence principle, reaching upper bounds of order  $10^{-12}$  [20, 21] for the free fall parameter  $\eta(A, B) = (a(A) - a(B))/g$ , where  $a(A)$  and  $a(B)$  are the accelerations of bodies  $A$  and  $B$  respectively and  $g$  a local reference gravitational acceleration. In this letter we analyze the local electromagnetic equations in the  $TH\epsilon\mu$  formalism, and show that there is an adiabatic non conservation of charge as measured in local experiments. We then analyze both a superstring based and a Bekenstein-like model on which we put stringent upper bounds on any violations of EEP several orders of magnitude tighter than any previous one. In this way, we obtain an effective test for string based and similar theories.

1: *Charge conservation in the  $TH\epsilon\mu$  formalism:*

Let  $S_{NG}$  be the action defining the  $TH\epsilon\mu$  formalism [2]:

$$\begin{aligned} S_{NG} = & - \sum_a m_{0a} \int (T - H v_a^2)^{1/2} dt \\ & + \sum_a e_a \int A_\mu v^\mu dt \\ & + \frac{1}{8\pi} \int (\epsilon E^2 - \mu^{-1} B^2) d^4x \end{aligned} \quad (1)$$

where  $T, H, \epsilon, \mu$  are functions of the spherically symmetrical gravitational potential  $\Phi(\mathbf{x})$ . It is assumed that these functions are slowly varying in the neighborhood of a given event  $\mathcal{P}$ , of the system  $NG$ . We shall choose a coordinate system with origin at  $\mathcal{P}$  and approximate these functions by linear functions of the local coordinates within the volume  $V$  of the system. So, in the neighborhood of  $\mathcal{P}$  we can expand the gravitational potential in the form:

$$\Phi(\mathbf{r}) = \Phi_0 + \mathbf{f}_0 \cdot \mathbf{x} + \dots \quad (2)$$

where  $\mathbf{f}_0$  is proportional to the local acceleration of gravity  $\mathbf{g}_0$ . In the same way we find  $T_0 + T_0' \mathbf{f}_0 \cdot \mathbf{x}$ , and similar expressions for  $H, \epsilon$  and  $\mu$ . Finally, we scale the coordinates in the form<sup>1</sup>:

$$\hat{t} = T_0^{1/2} t \quad \hat{\mathbf{x}} = H_0^{1/2} \mathbf{x} \quad (3)$$

Then, in the neighborhood of event  $\mathcal{P}$ , the action takes the form:

$$\begin{aligned} S_{NG} = & - \sum_a m_{0a} \int (1 - \hat{v}^2)^{1/2} d\hat{t} + \sum_a e_a \int \hat{A}_{\hat{\mu}} \hat{v}^{\hat{\mu}} d\hat{t} \\ & + \frac{1}{8\pi} \epsilon_0 \left( \frac{T_0}{H_0} \right)^{1/2} \int d^4 \hat{x} \hat{E}^2 \epsilon^*(\hat{x}) \\ & - \frac{1}{8\pi} \left( \frac{H_0}{T_0} \right)^{1/2} \mu_0^{-1} \int d^4 \hat{x} \hat{B}^2 \mu^{*-1}(\hat{x}) \\ & + \frac{1}{8\pi} \mathbf{f}_0 \cdot \int d^4 \hat{x} (\hat{\mathbf{E}} \times \hat{\mathbf{B}}) \sigma^*(\hat{x}) \end{aligned} \quad (4)$$

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<sup>1</sup>This scaling is a particular case of a more general transformation to a freely falling reference system, see references [2, 22].

where

$$\epsilon^* = \left( 1 + \frac{1}{2} \frac{T_0'}{T_0} \frac{\Gamma_0}{H_0^{1/2}} \mathbf{f}_0 \cdot \hat{\mathbf{x}} \right) \quad (5)$$

$$\mu^{*-1} = \left( 1 - \frac{1}{2} \frac{T_0'}{T_0} \frac{\Lambda_0}{H_0^{1/2}} \mathbf{f}_0 \cdot \hat{\mathbf{x}} \right) \quad (6)$$

$$\sigma^* = \epsilon_0 T_0' H_0^{1/2} \hat{t} \left( 1 - \frac{H_0}{T_0} \epsilon_0^{-1} \mu_0^{-1} \right) \quad (7)$$

and

$$\Gamma_0 = 2 \frac{T_0}{T_0'} \left( \frac{\epsilon_0'}{\epsilon_0} + \frac{1}{2} \frac{T_0'}{T_0} - \frac{1}{2} \frac{H_0'}{H_0} \right) \quad (8)$$

$$\Lambda_0 = 2 \frac{T_0}{T_0'} \left( \frac{\mu_0'}{\mu_0} + \frac{1}{2} \frac{T_0'}{T_0} - \frac{1}{2} \frac{H_0'}{H_0} \right) \quad (9)$$

As usual, electric and magnetic fields are related to the local scalar and vector potentials in the form:

$$\hat{\mathbf{E}} = \hat{\nabla} \hat{A}_0 - \hat{\mathbf{A}}_{,0} \quad \hat{\mathbf{B}} = \hat{\nabla} \times \hat{\mathbf{A}} \quad (10)$$

In equation 4, we shall make a final scaling:

$$e_a^* = T_0^{-1/4} H_0^{1/4} \epsilon_0^{-1/2} \quad A_{\hat{\mu}}^* = T_0^{1/4} H_0^{-1/4} \epsilon_0^{1/2} \quad (11)$$

which introduces the local particle charge  $e_a^*$ . Besides we introduce the local limiting velocity  $c_0$ , the local light velocity  $c_l$  and the ratio of both quantities  $c^* = c_l/c_0$ :

$$\begin{aligned} c_0 &= (T_0/H_0)^{1/2} \\ c_l &= (\epsilon_0 \mu_0)^{-1/2} \\ c^* &= (T_0^{-1} H_0 \epsilon_0^{-1} \mu_0^{-1})^{1/2} \end{aligned} \quad (12)$$

$$\begin{aligned} S_{NG} &= - \sum_a m_{0a} \int (1 - \hat{v}^2)^{1/2} d\hat{t} + \sum_a e_a^* \int \hat{A}^*_{,\hat{\mu}} \hat{v}^{\hat{\mu}} d\hat{t} \\ &+ \frac{1}{8\pi} \int d^4 \hat{x} \left[ \epsilon^* \hat{E}^{*2} - \mu^{*-1} \hat{B}^{*2} + \sigma^* \mathbf{f}_0 \cdot (\hat{\mathbf{E}}^* \times \hat{\mathbf{B}}^*) \right] \end{aligned} \quad (13)$$

Let us now introduce the local (renormalized) charge and current density:

$$\hat{\rho}^* = \sum_a e_a^* \delta(\mathbf{r} - \mathbf{r}_a) \quad (14)$$

$$\hat{\mathbf{j}}^* = \sum_a e_a^* \hat{\mathbf{v}}_a \delta(\mathbf{r} - \mathbf{r}_a) \quad (15)$$

Variation of (13) yields the inhomogeneous pair of Maxwell equations:

$$\begin{aligned} \hat{\nabla} \cdot (\epsilon^* \hat{\mathbf{E}}^* + \sigma^* \hat{\mathbf{B}}^* \times \mathbf{g}_0) &= 4\pi \rho^* \\ \hat{\nabla} \times (\mu^{*-1} \hat{\mathbf{B}}^*) &= \frac{\partial}{\partial \hat{t}} (\epsilon^* \hat{\mathbf{E}}^* + \sigma^* \hat{\mathbf{B}}^* \times \mathbf{g}_0) \\ &+ \hat{\nabla} \times (\sigma^* \mathbf{g}_0 \times \hat{\mathbf{E}}^*) \\ &+ 4\pi \hat{\mathbf{j}}^*. \end{aligned} \quad (16)$$

It is apparent that local conservation of charge still holds, as it is easy to derive the equation:

$$\hat{\nabla} \cdot \hat{\mathbf{j}}^* + \dot{\rho}^* = 0 \quad (17)$$

Thus, the locally conserved quantity is, in the  $TH\epsilon\mu$  formalism:

$$Q^* = \int_V \rho^* d^3\hat{x} \quad (18)$$

where the volume  $V$  is small in comparison with the scale of variation of the gravitational field  $V^{1/3} \ll L_g$ . For a system of identical particles, this may be written in the form:

$$Q^* = e^* N \quad (19)$$

Consider now an adiabatic change of  $e^*$ . Then the condition  $\dot{Q}^* = 0$  implies:

$$\frac{\dot{N}}{N} = -\frac{\dot{e}^*}{e^*} = \frac{1}{2} \left( \frac{\dot{\epsilon}_0}{\epsilon_0} + \frac{1}{2} \frac{\dot{T}_0}{T_0} - \frac{1}{2} \frac{\dot{H}_0}{H_0} \right) \quad (20)$$

and, using  $\dot{\epsilon} = \epsilon \dot{\Phi}$  and similar expressions, we find:

$$\frac{\dot{N}}{N} = \frac{1}{2} \left( \frac{\epsilon_0'}{\epsilon_0} + \frac{1}{2} \frac{T_0'}{T_0} - \frac{1}{2} \frac{H_0'}{H_0} \right) \dot{\Phi} \quad (21)$$

Now, from eq.2:

$$\dot{\Phi} = \mathbf{f}_0 \cdot \dot{\mathbf{x}} + \dots \quad (22)$$

and  $\mathbf{f}_0$  must be related to the local gravitational acceleration which is defined as the acceleration of a structureless particle. To find it, let us expand the first term of equation (1) in the neighborhood of  $\mathcal{P}$ . For a single uncharged particle we find:

$$S_P \simeq m_0 T_0^{1/2} \int \left( \frac{1}{2} \frac{v^2}{c_0^2} - \frac{1}{2} \frac{T_0'}{T_0} \mathbf{f}_0 \cdot \mathbf{x} \right) dt \quad (23)$$

The corresponding equation of motion is:

$$\frac{1}{c_0^2} \frac{d^2 \mathbf{x}}{dt^2} = \frac{1}{2} \frac{T_0'}{T_0} \mathbf{f}_0 \quad (24)$$

so we obtain, correct to the newtonian order:

$$\mathbf{f}_0 = \frac{T_0}{H_0^{1/2}} \frac{T_0}{T_0'} \frac{\hat{\mathbf{g}}_0}{c_0^2} \quad (25)$$

Finally, substitution in (22) and in (21) yields:

$$\frac{\dot{N}}{N} = \frac{1}{2} \frac{T_0}{T_0'} \left( \frac{\epsilon_0'}{\epsilon_0} + \frac{1}{2} \frac{T_0'}{T_0} - \frac{1}{2} \frac{H_0'}{H_0} \right) \hat{\mathbf{g}}_0 \cdot \hat{\mathbf{v}} \quad (26)$$

and introducing the local time (in seconds) through  $c_0 t^* = \hat{t}$  we obtain:

$$\frac{\dot{N}}{N} = \frac{\Gamma_0}{4} \frac{\mathbf{g}_0 \cdot \mathbf{v}}{c_0^2} \quad (27)$$

where the parameter  $\Gamma_0$ , defined in eq.8, characterizes anomalous accelerations and anomalous mass tensors [2]:

$$\delta m_P = 2\Gamma_0 \frac{E_C}{c_0^2} \quad (28)$$

$$\Delta a_C = \frac{\delta m_P}{m} g_0 \quad (29)$$

Equations (20) and (27) to (29) are the main result of this letter. They show that a breakdown of the weak equivalence principle by electromagnetic interactions and conservation of charge are not independent. Furthermore, the conservation of  $Q^*$  implies that there is a current of neutral particles, carrying out particle number, from the decay of the charged ones.

**2: Extension to superstring theories:** The main result obtained in the previous section can be extended to some cases of superstring theories, namely those with a massless dilaton. Let us concentrate in the matter action of the model proposed in ref.[23]

$$S_m = - \int d^4x \sqrt{g} [-\psi \gamma^\mu (\partial_\mu - i A_\mu) \psi] - \int d^4x \sqrt{g} \frac{1}{4} k B_f(\phi) F^{\mu\nu} F_{\mu\nu} \quad (30)$$

The third term of last equation accounts for the electromagnetic field contribution with  $\epsilon = \mu^{-1}$ . Furthermore, we can identify the second term with the coupling between matter and electromagnetism, and the first one with the kinetic contribution with  $H = T = 1$ .

Thus, eq.20 holds and since the charge measured in the free falling system is  $e^*$ , we can write the following equation:

$$\frac{\dot{\alpha}}{\alpha} = 2 \frac{\dot{e}^*}{e^*} = -2 \frac{\dot{N}}{N} \quad (31)$$

It can be shown [23] that in this model the following relation exists between the the anomalous acceleration  $\Delta a_C$  and the cosmological variation of  $\alpha$ :

$$\frac{\Delta a_C}{a} \sim 10^{-2} \frac{\dot{\alpha}}{\alpha H_0} = \frac{2 \times 10^{-2}}{H_0} \frac{\dot{N}}{N} \quad (32)$$

which is peculiar to this model.

### 3: Bekenstein-like theories

In order to study the fine structure constant variability, Bekenstein [19] proposed a theoretical framework based on very general assumptions. In this context, every particle charge can be expressed in the form  $e = e_0 \varphi(\vec{x}, t)$  where  $\varphi(\vec{x}, t)$  is a scalar field and the matter action of a system of particles can be written as follows:

$$S_{NG} = - \frac{1}{16\pi} \int \varphi^{-2} F^{\mu\nu} F_{\mu\nu} d^4x + \sum_i \int \left[ mc^2 + \frac{e_0}{c} u^\mu A_\mu \right] \gamma^{-1} \delta^3(x^i - x^i(\tau)) d^4x \quad (33)$$

where  $A_\mu$  is  $\varphi$  times the gauge field as defined by Bekenstein.

Thus, we can identify the first term with the electromagnetic contribution in the  $TH\epsilon\mu$  formalism with  $\epsilon = \mu^{-1} = \varphi^{-2}$  and the second term with the matter and coupling between matter and electromagnetism with  $T = H = 1$ . For  $N$  identical particles we have:

$$\frac{\dot{N}}{N} = - \frac{\dot{\varphi}}{\varphi} = - \frac{1}{2} \frac{\dot{\alpha}}{\alpha} \quad (34)$$

where  $\alpha$  is the fine structure constant. It is easy to show that a relation similar to eq.32 holds in this model. Using the results in ref.[19] we obtain:

$$\frac{\Delta a_C}{a} \sim 2 \times 10^{-3} \frac{\dot{\alpha}}{\alpha H_0} = \frac{4 \times 10^{-3}}{H_0} \frac{\dot{N}}{N} \quad (35)$$

Process	Ref.	$\tau$ (y)	$\frac{\dot{\alpha}}{\alpha} (y^{-1})$	$\frac{\Delta a}{a}  _{DP}$	$\frac{\Delta a}{a}  _{Beck}$	$\frac{\Delta a}{a}  _{TH\epsilon\mu}$
$^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$	[25]	$4 \times 10^{26}$	$6 \times 10^{-27}$	$10^{-18}$	$2 \times 10^{-19}$	$2 \times 10^{-13}$
$e \rightarrow \nu_e \gamma$	[26]	$2 \times 10^{25}$	$8 \times 10^{-26}$	$10^{-17}$	$2.5 \times 10^{-18}$	$2 \times 10^{-11}$
$e \rightarrow \text{any}$	[27]	$3 \times 10^{23}$	$8 \times 10^{-24}$	$10^{-15}$	$4 \times 10^{-16}$	$2 \times 10^{-9}$
$e \rightarrow \nu_e \gamma$	[28]	$2 \times 10^{26}$	$5 \times 10^{-27}$	$2 \times 10^{-18}$	$3 \times 10^{-19}$	$2 \times 10^{-12}$
$e \rightarrow \text{any}$	[29]	$2 \times 10^{24}$	$10^{-24}$	$5 \times 10^{-16}$	$3 \times 10^{-17}$	$2 \times 10^{-10}$

Table 1: Results. The columns show the process considered, the corresponding references, the observational data, the limits on the time-variation of the fine structure constant and the bounds for the breakdown of the equivalence principle obtained from Damour and Polyakov-like theories, Bekenstein-like models and the general relationship derived in this paper.

#### 4: Comparison with experiments

There have been many experiments to put bounds on processes that change charge discontinuously, such as the disappearance of electrons [24]. Thus, we can use these results to put bounds on  $\alpha$  variation and can use the relation between  $\alpha$  variation and  $\Delta a/a$  of eq.32 to put bounds on the breakdown of the equivalence principle. Similar results, somewhat stronger, hold for Bekenstein-like models, improving the limits established in ref.[19] on violation of EEP. Results are shown in Table 1.

When we use our relations (27) to (29) we have

$$\frac{\Delta a_C}{a} = 8 \frac{\dot{N}}{N} \frac{E_c}{m c_0^2} \frac{c_0^2}{g_0 \cdot v}. \quad (36)$$

For the fall towards the Virgo Cluster we estimate  $v/c_0 \simeq 10^{-3}$ ,  $g_0/c_0^2 \simeq 10^{-16} m^{-1}$  and typically  $E_c/mc_0^2 \simeq 10^{-3}$  and we obtain

$$\frac{\Delta a_C}{a} = 3 \times 10^{14} y \frac{\dot{N}}{N} \leq 10^{-12} \quad (37)$$

which is a much weaker bound than the one using Damour and Polyakov model relationship and even more weak than the limit obtained from the Bekenstein model relationship.

This can be understood as follows: the bound from expression (27) comes from the anomalous coupling of the electromagnetic energy with gravity, while the bound from expression (32) comes from the dilaton exchange mechanism as used in [23], which is a much more strong effect than the electromagnetic one. On the other hand, expression (35) comes from the close link between the gradient of the gravitational potential and the gradient of  $\alpha$  in Bekenstein theory, which we do not consider in our adiabatic  $TH\epsilon\mu$  treatment.

We see then that there is a deep connection between charge non-conservation and violation of universality of free fall for a wide class of theories, namely those that can be written in the  $TH\epsilon\mu$  form. The connection, as expressed in (27), is considerably general, and provides a link between any electromagnetic violation of EEP and non conservation of charge. The corresponding bounds on WEP are comparable to present day values. The connection (32) is more specific from dilaton-type theories, a special case of  $TH\epsilon\mu$  theories, which are those that provide the mechanism considered in [23]. In this case the bounds obtained are even lower than proposed future direct tests of WEP [21]. Consequently these future tests still deserve much attention, though they may add not too much new information as regards to dilaton-type gravitational theories, as long as the local coupling of the dilaton field can be neglected.

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